A Concave FE-BI-MLFMA for Scattering by a Large Body with Nonuniform Deep Cavities

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***Abstract*— A concave finite element-boundary integral-multilevel fast multipole algorithm (FE-BI-MLFMA) is presented for scattering by a large body with nonuniform deep cavities. Different from the conventional FE-BI-MLFMA, the boundary integral equation in this concave FE-BI-MLFMA is established on a concave surface to reduce the region of the finite element method (FEM), which can significantly reduce dispersion error from the FEM and improve efficiency of FE-BI-MLFMA especially for nonuniform cavities. To eliminate the problem of slow convergence caused by concave surface, an efficient preconditioner based on the sparse approximate inverse (SAI) is constructed in this paper. Numerical experiments demonstrate the accuracy and efficiency of this SAI preconditioned concave FE-BI-MLFMA for nonuniform deep and large cavites. This SAI preconditioned concave FE-BI-MLFMA is parallelized to further improve its capability in this paper. An extremely big and deep nonuniform cavity has been calculated, demonstrating the great capability of this parallel concave FE-BI-MLFMA.**

I. INTRODUCTION

A layer-based FE-BI-MLFMA has been successfully developed to compute scattering by a large body with big and deep cavities [1][2]. Our recent study shows that the algorithm of the FE-BI-MLFMA in [2] still has some challenge problems for real applications. The dispersion error is big when the FEM is employed for modeling large and deep cavities in real applications. More importantly, the cavities in real applications such as jet engine inlet are usually nonuniform, the layer-based FE-BI-MLFMA in [2] becomes inefficient for this kind of nonuniform cavities. To eliminate these problems, a concave FE-BI-MLFMA is presented in this paper. In the conventional FE-BI-MLFMA, the integral equation is established on a convex surface, thus the cavity has to be whole modeled by the FEM. In the proposed concave FE-BI-MLFMA, the integral equation is established on a concave surface, only a small nonhomogeneous or complex part of the cavity need to be modeled by the FEM. The other empty part of the cavity is modeled by the boundary integral equation. Since the FEM modeling is restricted in a small region in this concave FE-BI-MLFMA, the dispersion error and inefficiency caused by the FEM can be avoided. However, since the integral equation in the proposed FE-BI-MLFMA is established on a concave surface, the condition number of the final discretized matrix equation usually becomes worse. To overcome this problem, an efficient preconditioner based on the SAI is constructed for improving the efficiency of our concave FE-BI-MLFMA. Numerical experiments demonstrate great advantages of this concave FE-BI-MLFMA in accuracy and efficiency over the conventional convex FE-BI-MLFMA especially for nonuniform deep and large cavites often encountered in real applications.

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II. FORMULATION OF CONCAVE FE-BI-MLFMA

Consider scattering by an arbitrarily shaped body with a deep cavity, as shown in Fig. 1. According to the FE-BI-MLFMA presented in [2], the solution region is directly divided into the interior region and the exterior region. The interior region is the cavity region, denoted as V_i , bounded by the inner wall of the cavity S_i and the aperture surface S_c. The exterior region is free space outside S_o and S_c . In the conventional FE-BI-MLFMA [10], the chosen surface S_c usually is the opening surface of the cavity, as shown in Fig.1(a). Thus the boundary surface constituted by S_i and S_c is convex. The interior region enclosed by this boundary surface is extremely large for big and deep cavities. Unlike this conventional FE-BI-MLFMA, the surface S_c is chosen as shown in Fig.1(b). Thus the boundary surface constituted by S_i and S_c is concave. The interior region enclosed by this concave surface can be very small even for large and deep cavities.

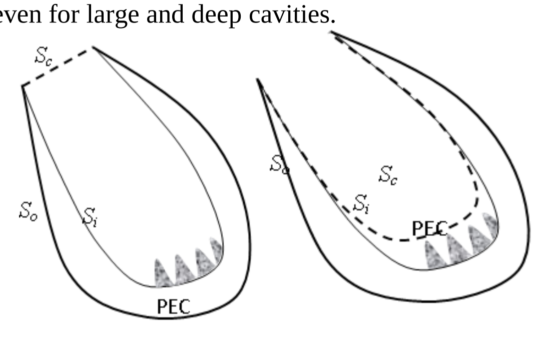


Fig. 1. Scattering by an arbitrarily shaped body with a deep cavity
(a)Convex Surface (b) Concave Surface

Similar to the conventional FE-BI-MLFMA, the higher-order curvilinear vector finite element is employed to model the interior region of the cavity, and the zeroth-order vector basis function, namely the RWG (Rao-Wilton-Glisson) one, is employed to model complex exterior region of the cavity. To be more specific, the fields in the interior region can be formulated into an equivalent variational problem with the functional. Using higher-order vector basis functions discretizing the functional yields :

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ic} & 0 \\ \mathbf{K}_{ci} & \mathbf{K}_{cc} & \mathbf{B}_{cc} \end{bmatrix} \begin{Bmatrix} E_i \\ E_c \\ H_c \end{Bmatrix} = \{0\} \tag{1}$$

where $\{E_c\}$, $\{H_c\}$ denotes the unknowns on the chosen surface S_c , and $\{E_i\}$ denotes the unknowns in the interior cavity. The fields in the exterior region can be formulated into the combined field integral equation (CFIE). Using the RWG basis function [13], the CFIE can be discretized as follows [10]:

$$\begin{bmatrix} \mathbf{P}_{cc} & \mathbf{Q}_{cc} & \mathbf{Q}_{co} \\ \mathbf{P}_{oc} & \mathbf{Q}_{oc} & \mathbf{Q}_{oo} \end{bmatrix} \begin{Bmatrix} E_c \\ H_c \\ H_o \end{Bmatrix} = \begin{Bmatrix} b_c \\ b_o \end{Bmatrix} \tag{2}$$

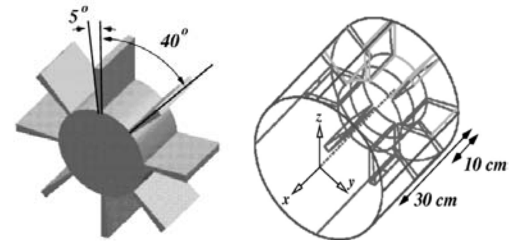
The final equation system can be generated by combining (1) with (2). This equation system is hard to be solved because of its poor condition number. To solve it efficiently, the relation of $\{E_c\} = [\mathbf{M}]\{H_c\}$ is first to be obtained by applying the layer-based fast direct solver to the FEM matrix equation. Then, substituting this relation of $\{E_c\} = [\mathbf{M}]\{H_c\}$ into (2) yields:

$$\begin{bmatrix} \mathbf{P}_{cc}\mathbf{M} + \mathbf{Q}_{cc} & \mathbf{Q}_{co} \\ \mathbf{P}_{oc}\mathbf{M} + \mathbf{Q}_{oc} & \mathbf{Q}_{oo} \end{bmatrix} \begin{Bmatrix} H_c \\ H_o \end{Bmatrix} = \begin{Bmatrix} b_c \\ b_o \end{Bmatrix} \tag{3}$$

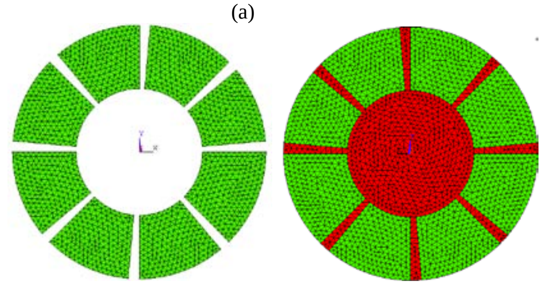
In the conventional FE-BI-MLFMA, (3) usually has a better condition number. Thus, (3) can be efficiently solved with iterative solvers such as GMRES (Generalised Minimal Residual) by employing MLFMA to speed up the matrix-vector multiplication. However, our numerical experiments show that the condition number of (3) in our proposed FE-BI-MLFMA become worse due to concave surface. To overcome this problem, we need to construct a preconditioner. There are two efficient preconditioners for boundary integral equations. One is the incomplete LU preconditioning (ILU), the other is the sparse approximate inverse preconditioning (SAI). Since the SAI is more suitable for parallel implementation, we here adopt the SAI preconditioning for efficiently solving (3).

III. Numerical Results

To demonstrate the accuracy and efficiency of the proposed concave FE-BI-MLFMA described above, we compute scattering by a circular cavity loaded by an array of straight blades, as shown in Fig.2. This example was computed by the convex FE-BI-MLFMA in [3]. Here we employ a concave boundary surface, whose bottom part in the cavity is the top face of the metallic blades. This choice of the concave boundary surface greatly reduces the number of unknowns in the FEM regions. The maximum number of unknowns on each layer face can be reduced from 20632 to 14400 by about 30%, which greatly improves efficiency and reduce memory requirement of the FE-BI-MLFMA to about half of those in [3]. Figure 3 presents the comparison of VV-polarized monostatic RCS for this complex cavity between numerical results and measured data.



(a)



(b)

Fig.2 (a).Geometry of a circular cavity loaded by an array of straight blades (b) Finite element mesh

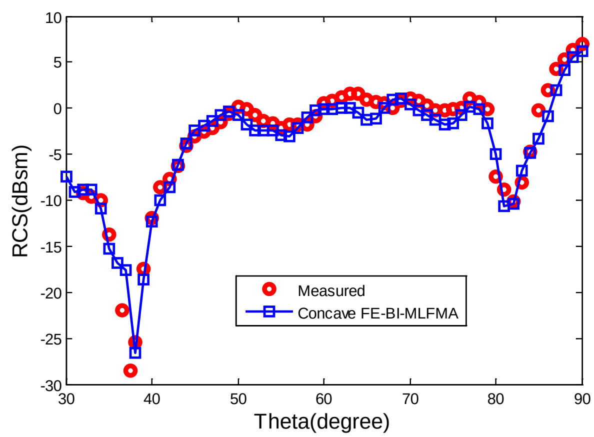


Fig. 3 Monostatic RCS of a circular cavity loaded with an array of blades at 6GHz (VV-Polarization)

References

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